Duration and Interest Rate Sensitivity of Bank Portfolios

Consider the following bank balance sheet. Interest rates are fixed rates on pure discount securities. Interest rates are 6 percent on liabilities and 8 percent on assets.

Bank Balance Sheet (Duration in years in parenthesis)

($ millions)

Assets	$million	Duration Years	Liabilities and Equity	$million	Duration Years
Prime-Rate Loans (rates set monthly)	20	**(1.0)**	Super Now Checking Accounts (rates set monthly)	50	**(1.0)**
2-Year Car Loans	40	**(1.0)**	6-Month Certificates of Deposit	20	**(0.5)**
30-Year Mortgages	40	**(7.0)**	3-Year Certificates of Deposit	20	**(3.0)**
			Liabilities	90	**(1.33)**
			Equity	10	
Assets	**100**	**(3.4)**	**Liabilities and Equity**	**100**	**(3.4)**

$$D_{portfolio} = \frac{\sum_{j=1}^{k} X_j D_j}{\sum_{j=1}^{k} X_j}$$

, **where X_j is the value of each of the k portfolio components.**

Duration of Assets: D_A **= [20*(1.0)+40*(1.0)+40*(7.0)]/(100) = 3.40**

Duration of Liabilites: D_L **= [50*(1)+20*(0.5)+20*(3.0)]/(90) = 1.33**

Definitions

Δ E	**=**	**change in market value of the portfolio,**
DA	**=**	**duration of assets,**
DL	**=**	**duration of liabilities,**
E	**=**	**market value of equity,**
L	**=**	**market value of liabilities,**
A	**=**	**market value of assets, and**
Δ y	**=**	**change in interest rates.**

$$\Delta E = -\Delta y\left[\frac{D_A}{(1+y_A)} - \frac{L}{A}\frac{D_L}{(1+y_L)}\right]A$$

$$\Delta A = -\Delta y \frac{D_A}{(1+y_A)} A, \Delta L = -\Delta y \frac{D_L}{(1+y_L)} L$$

Change in Equity with a 200 bp increase in all interest rates:

Δ E = -(.02)*[(3.4/1.08) - (0.9)*(1.33/1.06)]*100 = -4.04 = $ -4.04 million

Δ **A = -(.02)*(3.4)(100)/(1.08) = $ -6.30**

Δ **L = -(.02)*(1.33)*(90)/1.06) = $ -2.26**

Immunization of Equity to Changes in Interest Rates

Setting Δ E = 0 implies:

$$\frac{D_A}{(1+y_A)} = \frac{L}{A}\frac{D_L}{(1+y_L)}$$

At the stated interest rates and duration of liabilities, immunization implies a D_A of:

$$\frac{\Delta E}{A} = -\Delta y\left[\frac{D_A}{(1+y_A)} - \frac{L}{A}\frac{D_L}{(1+y_L)}\right]$$

D_A = [(0.9)*(1.33*1.08)/(1.06)] = 1.22 years

NOTE: A simple matching of Duration would imply a D_A of 1.33 years.

However, this strategy ignores relative interest rates and leverage.