Research Task Group

Introductory Seminar
Fundamentals of Derivative Instruments

Pricing Fixed/Floating Swap Contracts
As Implied Forward Contracts

Pricing Interest Rate Option-Like Contracts
Caps, Floors and Collars

and

Measuring Risk of a Portfolio

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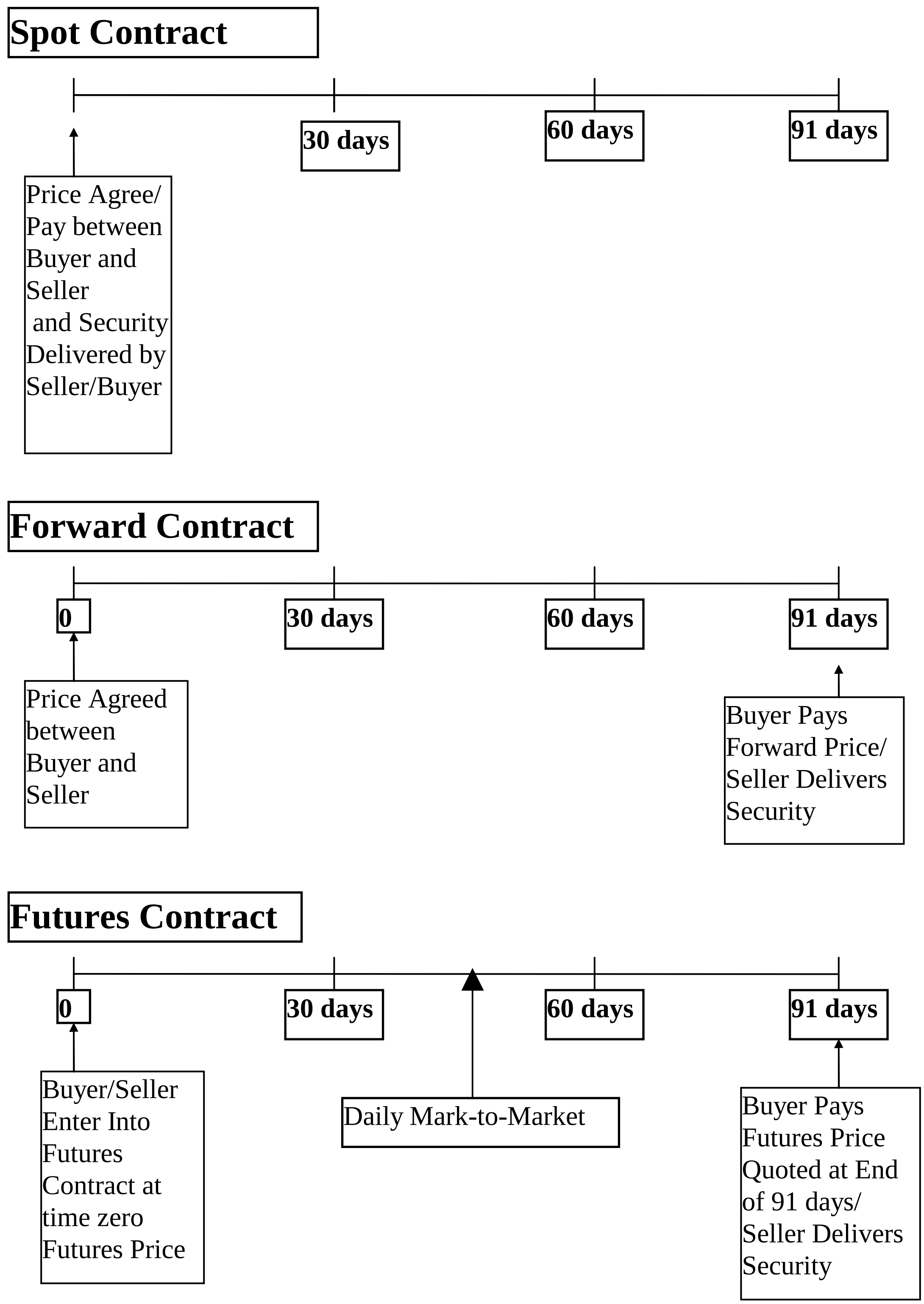
Spot, Futures and Forwards Contract Time Lines



Treasury Bill and Eurodollar Pricing Conventions

Priced on a Discount Yield Basis and 360 Day Year

P = Price $\quad$ **y_d = discount yield (annual rate, in decimal)**

F = Face value ($100) $\quad$ **dtm = days to maturity**

T-Bills have original maturities of 91, 182 and 364 days

<u>Discount Price</u>

$$P = F(1-y_d(dtm/360)) = 100 - 100\, y_d(dtm/360)$$

Note the futures market price quotes ignore the maturity conversion (dtm/360), giving the futures price as $P=100-Y_d$(annual, percent).

<u>Discount Yield (Return on the Face Value)</u>

$$y_d = \left(\frac{F - P}{F}\right)\frac{360}{dtm}$$

<u>Example</u>

November 28, 1994 T-bill Auction

<u>91 Day T-bill</u>	**<u>182 Day T-bill</u>**
$y_d[91] = 5.44\ \%$	$y_d[182] = 5.86\ \%$
$P = 100(1 - (0.0544)(91/360))$	$P = 100(1 - (0.0586)(182/360))$
<u>P = 98.625</u>	**<u>P = 97.037</u>**

Computing Rate of Return - also known as Money Market Yield

$$\mathbf{r} = \left(\frac{\mathbf{F - P}}{\mathbf{P}}\right)\frac{\mathbf{360}}{\mathbf{dtm}} = \left(\frac{\mathbf{F}}{\mathbf{P}} - \mathbf{1}\right)\frac{\mathbf{360}}{\mathbf{dtm}}$$

This can be simplified by letting F=1:

$$\mathbf{r} = \left(\frac{\mathbf{1}}{\mathbf{P}} - \mathbf{1}\right)\frac{\mathbf{360}}{\mathbf{dtm}}$$

In terms of a rate of return factor in units of the dtm time period:

$$\left(\mathbf{1} + \mathbf{r}\frac{\mathbf{dtm}}{\mathbf{360}}\right) = \frac{\mathbf{1}}{\mathbf{P}}$$

Example

November 28, 1994 T-bill Auction

91 Day T-bill	**182 Day T-bill**
P = 98.625	**P = 97.037**

$$r[91] = \left(\frac{100 - 98.625}{98.625}\right)\frac{360}{91} \qquad r[182] = \left(\frac{100 - 97.037}{97.037}\right)\frac{360}{182}$$

$$r[91] = 0.0552 \qquad r[182] = 0.0604$$

Compare with the T-bill discount yields:

91 Day T-bill	**182 Day T-bill**
$\mathbf{y_d[91] = 0.0544}$	$\mathbf{y_d[182] = 0.0586}$

Returns are useful in comparing the results of different investment alternatives and in computing forward returns and converting them to discount yields on T-bills and futures prices.

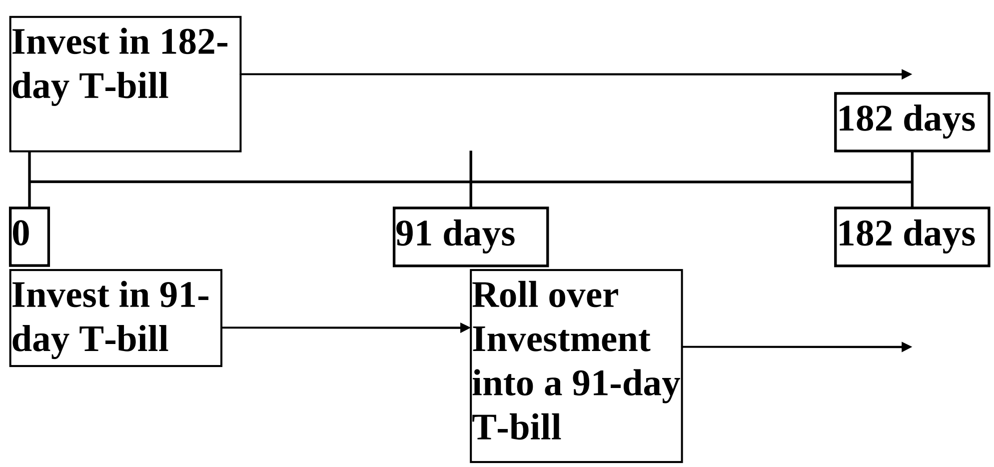
<u>Understanding Implied Forward Yields</u>
<u>Example Using T-bills</u>

A forward contract on a 91-day T-bill is an obligation to buy or deliver, at the expiration of the contract, a T-bill with 91 days to maturity. At expiration, the buyer (long) of a T-bill forward can buy the T-bill with 91 days to maturity at the forward price quoted when the contract was initiated, and the seller (short) must deliver the T-bill at this same price.

At any time, investors interested in T-bill investments face expectations of what a T-bill with 91 days to maturity will be worth in the future.

For example, consider a banker with an investment horizon of 182 days. The banker has the opportunity to invest in a T-bill of 182 days to maturity or a 91 day T-bill maturing in 91 days and, upon maturity, rolling over the proceeds of this investment into a 91-day T-bill.

<u>Time Line for 182-day T-bill compared with two 91-day T-bills</u>



Understanding Implied Forward Yields (continued)

Which investment should the banker make?

Given they are both default free, the banker will be indifferent between them if the value of the investments at the end of the investment horizon are the same or the total returns from the investments are equal. These conditions imply that:

$$(1+r[182](182/360)) = (1+r_0[91](91/360))(1+E(r_{91}[91])(91/360))$$

$r_0[182]$ = the return on a 182-day T-bill at the end of the initial period.

$r_0[91]$ = the return on a 91-day T-bill bill at the end of the initial period.

$E(r_{91}[91])$ = the expected return on a 91-day T-bill at the end of the first 91 days and with 91 days to maturity.

Assuming the expected future 91-day T-bill return will on average equal the actual 91-day T-bill return (the expectations hypothesis), this expectation can be computed from the returns implied by the yield curve at time zero.

Rearranging the above equation gives the Implied Forward Return as:

$$r_{91}[91](91/360) = [(1+r_0[182](182/360)) / (1+r_0[91](91/360))] - 1$$

Example

From the previous example: $r_0[182]=0.0604$ and $r_0[91]=0.0552$. The implied forward rate of return is:

$r_{91}[91](91/360) = 0.01636$ and $r_{91}[91] = 0.06472$ in annual rate.

Convert the Implied Forward Return on a T-bill to a T-bill Yield

To compare the forward return with a T-bill yield and a futures yield, the implied forward return must be converted to the discount yield quoted on T-bills.

Let $y_{91}[91]$ be the implied forward 91-day discount yield in 91 days. This can be computed from the implied 91-day forward return as:

$$y_{91}[91](91/360) = 1 - 1/(1+r_{91}[91](91/360))$$

Example - Data From above

$$r_{91}[91](91/360) = 0.01636$$

$$y_{91}[91](91/360) = 1 - 1/(1.01636) = 0.0161 \text{ (91-day rate)}$$

In terms of annual rate:

$$y_{91}[91] = 0.06369 \text{ or } 6.369 \text{ percent.}$$

Comparison of current yields and Implied Forward Yields as of the November 28, 1994 Auction

Current 91-day $y_0[91]$	Current 182-day $y_0[182]$	Implied Forward $y_{91}[91]$	Futures Yield (11/29/94) $y_F[91]$ (Mar 95)
5.44 %	5.86 %	6.37 %	6.35 %

The implied forward can be interpreted as the market's expectation of the yield of a 91-day T-bill in the first week of March 1995. This implies a significant increase from the current 91-day T-bill yield of 5.44 percent of 93 bp. However, the futures market yield for the Mar 95 contract is lower, suggesting the future is more expensive than the market implied forward price (sell future and buy forward).

Example: Long Futures Position

On January 2 a banker expects to receive $1 million in a CD deposit in mid-March which will be invested in a 91-day T-bill. If interest rates decline, the banker will forego return compared with today's 91-day T-bill yield of 5.68 percent and a price of 94.32. However, if interest rates rise, the banker will earn more than today's yield. To guard against an interest rate decline, the banker can buy a T-bill future for delivery of a 90-day T-bill in March at about the time the funds will be available.

January 2

Buy T-bill future at 93.55 implying a discount yield of 6.45 %.
Current 91-day T-bills due in March are yielding 5.68 %.

March 18 - Interest rates fell from 5.68 % to 5.00 %

Future expires at 95.00 yielding 5.00 %.
90-day T-bills coming due in June selling for 95.00.
Take delivery, or close position by selling the contract.

Futures Profit:	**Selling Price**	**95.00**
	Purchase Price	**93.55**
	Profit	**1.45**

At $25 per bp, profit is $3,625.

Opportunity Loss:	**Target Price**	**94.32**
	Current Price	**-95.00**
	Loss	**-0.68**

At $25 per bp, loss is $1,700.

Thus, the banker gained by taking a long position in the futures market and hedging against an interest rate decline.
NOTE: if rates had risen to future rate of 6.45 %, no gain in futures.

Pricing Fixed/Floating Swap Contracts As Implied Forward Contracts

Generic Pricing Policy for "Plain Vanilla" Fixed/Floating Swaps:

1. Swaps are priced at par, where par is the notional amount.

2. The "fair" fixed and floating rates are set such that expected present values (PV) of the cash flows are the same for both counterparties.

Expected PV(Fixed) = Expected PV (Floating)

Discount factor for each period T for each element of PV(Floating) = $\prod_{t=1}^{T} \frac{1}{\left(1+\frac{F_t}{f}\right)}$,

where F_t is the zero-coupon forward rate from the yield curve adjusted for the markup over the corresponding Treasury at an annual rate, *f* is the frequency of reset (1 for 1 year, 2 for six-months, etc), and *t* is the time index. For example, if F_1 is 5 percent and F_2 is 6 percent and the frequency of reset is 6-months, the discount factor for a cash flow in the first period is $\frac{1}{\left(1+\frac{.05}{2}\right)}$ and for the second period is $\left[\frac{1}{\left(1+\frac{.05}{2}\right)}\right]\left[\frac{1}{\left(1+\frac{.06}{2}\right)}\right]$.

(Refer to pages 586-596, Fabozzi and Modigliani, 3rd edition for an example.)

3. If the swap is not priced as in point 2, upfront payments are made by either party to ensure this "no arbitrage" condition.

4. Swaps are generally priced off of the Treasury bond par yield curve. This means that the floating rates are derived from the Treasury bond par yield curve to develop implied zero-coupon forwards to value the floating coupons (see T-bill example above).

The Treasury Bond Par Yield Curve is the yield curve based on T-bonds selling at par ($100) -- usually those most recently issued. It can also be constructed since T-bonds are usually issued at par.

Example Terms of a "Plain Vanilla" Swap Agreement:

Notional :	**$200 million**
Maturity:	**5 years, originating November 4, 1993**
Reset Dates:	**6 months, beginning in 6 months (May 4, 1994)**
Fixed Rate:	**5.30 percent (based on 30-day month and 360-day year)**

FloatingRate: Commercial Paper (CP) 30-day

On November 4, 1993, the 5-year, T-bond, constant maturity Treasury (CMT) was 5.033 percent (Chart 4) and the 5-year swap rate on that date was 5.288 percent, a 25.5 bp swap-spread over the T-bond rate. The implied 6-month forward rate quoted on Eurodollars (approximating the CP) was 3.80 percent and the spot 30-day CP rate was 3.15 percent.

Pricing Fixed/Floating Swap Contracts As Implied Forward Contracts (Continued)

Expected Cash Flows (%) From the Swap as of November 4, 1993 (Approximate and based on Eurodollar (ED) forward quotes)

Reset Date	Fixed Rate (%)	Implied Forward Floating Rate (%)	Difference (%)
May 4, 1994	5.3	3.8	1.5
November 4, 1994	5.3	4.5	0.8
May 4, 1995	5.3	4.9	0.4
November 4, 1995	5.3	5.2	0.1
May 4, 1996	5.3	5.5	-0.2
November 4, 1996	5.3	5.9	-0.6
May 4, 1997	5.3	6.1	-0.8
November 4, 1997	5.3	6.4	-1.1
May 4, 1998	5.3	6.5	-1.2
November 4, 1998	5.3	6.4	-1.1

The implied forward floating rate is the expectation of the spot rate that will prevail at each of the dates.

CRITICAL ASSUMPTIONS IN PRICING Fixed/Floating SWAPS

1. All of those listed in 1-4 above.
2. Implied forward, zero-coupon interest rates can be derived from the Treasury par yield curve.
3. <u>Implied forward rates are unbiased estimates of future spot rates.</u> This assumption also implies that the volatility of forward interest rates (forward volatility) is constant.
4. Default risk is constant at each period of the swap.

Finding the Swap Rate given the forward/futures rates:

Swap Rate = PV(floating rate cash flows)/[Notional*(1/frequency)* $\sum_{i=1}^{T} FwdDcntFac\, i$]

Pricing Interest Rate Option-Like Contracts
Caps, Floors and Collars

Definitions

Cap: **A cap is a European call option on interest rates frequently with multiple exercise dates. The buyer of a cap has the right, but not the obligation, to receive an interest payment on a stated notional amount at the end of each cap reset or exercise date of an amount of interest equal to the difference between the market or index rate and the strike rate times the notional amount. If the market or index rate is at or below the strike rate, the cap buyer receives nothing. The seller of a cap, agrees to pay the buyer. The buyer pays the seller a premium for this right. (See Figures 4 and 5 for graphs of the payoff of a one period cap.)**

Floor: **A floor is a European put option on interest rates frequently with multiple exercise dates. The buyer of a put has the right to receive an interest payment on a stated notional amount at the end of each floor reset or exercise date of an amount of interest equal to the difference between the strike rate and the market or index rate times the notional amount. If the market or index rate is at or above the strike rate, the floor buyer receives nothing. The seller of a floor, agrees to pay the buyer. The buyer pays the seller a premium for this right. (See Figure 6 for a graph of the payoff of a one period floor.)**

Collar: **A collar is the taking of a position simultaneously in a cap and a floor. For example, buying a collar is the same as buying a cap and selling floor where the buyer pays a premium for the cap and receives a premium for the sale of the floor. (See Figure 7 for a graph of the payoff of a one period collar purchase.)**

Pricing Interest Rate Option-Like Contracts (continued)

Generic Pricing of a Cap With Multiple Exercise Dates

Principles:

1. **Consider the option at each exercise date as a separate cap -- a caplet.**
2. **Price each caplet and sum the properly discounted prices to arrive at the price of the entire cap.**
3. **At each exercise date, the realized rate can be expressed as the forward rate for the underlying instrument or index.**

Pricing Approaches and Models

1. Use Black's options pricing model (F. Black, *Journal of Finance*, March 1976, and Hull, 1993, p373-378) to price each caplet assuming:

- **a. Constant volatility of forward interest rates**
- **b. Interest rate changes are normally distributed**
- **c. The risk-free rate is constant, though the underlying rate is stochastic.**

Black's (1976) Model for Pricing Each Caplet

Definitions: L = the notional amount.

- **f = the frequency of reset (1 for annual, .5 for 6-month, etc.)**
- **k = the current period.**
- **r = the risk-free rate for an instrument that matures in at time kf.**
- **R_X = the exercise rate.**
- **R_k = the observed underlying rate at period k.**
- **F_k = forward rate on the underlying interest rate.**
- **σ_F = the standard deviation of the change in the underlying rate.**
- **c,p = price of the cap (c) and price of the put (p).**

Recognizing the payoff at each caplet period k, discounted by the forward interest rate factor (end of period payment) is:

$$payoff = \frac{fL}{1 + fF_k} \max(R_k - R_X, 0)$$

Pricing Interest Rate Option-Like Contracts (continued)

Price of the cap:

$$c_k = \frac{fL}{1 + fF_k} e^{-rkf} \left[F_k N(d_1) - R_X N(d_2)\right] \text{, where}$$

$$d_1 = \frac{\ln(F_k / R_X) + 0.5\sigma_F^2 kf}{\sigma_F \sqrt{kf}}$$

$$d_2 = d_1 - \sigma_F \sqrt{kf}$$

NOTE:

1. The price of the caplet at period k is positively related to the volatility of interest changes on the underlying interest rate. Thus a higher volatility, σ_F, means that the value of each caplet and the entire cap will also be greater -- the likelihood that R_k will exceed R_X increases.

2. The assumption is made again that the forward rate is an unbiased estimate of the future spot rate.

3. The value of the entire cap is found by summing the values of the caplets since each is properly discounted back to the present.

2. <u>Black's (1976) Model treating a cap as a portfolio of put options on zero-coupon discount bonds</u>.

This approach views the option as a put option on a single period zero-coupon bond where the period of maturity is equal to the reset period.

Pricing Interest Rate Option-Like Contracts (continued)

3. Binomial and Multinomial Tree Approaches to Pricing Options

These approaches begin by assuming:

1. A risk-free arbitrage can be established in markets.

2. The interest rate term structure in a risk-neutral world is determined by a stochastic process that depends only upon the short-term rate of interest -- one factor models. Such a diffusion process underlying the models is:

$$dr = m(r)dt + \sigma(r)dz ,$$

where, dr is the continuous change in the short-term interest rate, m(r) is the mean reversion or drift term, σ(r) is the volatility of interest rate changes, and dz is a Wiener process.

Employing such models requires estimating the parameters of the model such that they fit the current term structure and known market prices of bonds and options. Parameters need not be constant over all time periods.

For the one factor models, the term structure can take on any form, interest rates changes are in the same direction for all terms, and interest rate changes need not be of the same amount over the entire term structure.

Discount bond prices and interest rates can be evaluated with these term structure models.

Complex European and American options and complex interest rate derivatives can be evaluated using these models.

Binomial tree structures can yield rich results, but they do not allow for interest rates or bind prices to take on a no change value.

Pricing Interest Rate Option-Like Contracts (continued)

A Real World Example of a Complex European Option with a Swap

5 Year/30 Year Linked Swap between Procter & Gamble, Company (P&G) and Bankers Trust Company (BT)

Terms:

Notional :	**$200 million**
Maturity:	**5 years, originating November 4, 1993**
Reset Dates:	**6 months, beginning in 6 months (May 4, 1994)**
BT Pays:	**5.30 percent (based on 30-day month and 360-day year)**
P&G Pays:	**Period 1: Commercial Paper (CP) 30-day minus .75%**
	Period 2-10: CP - .75% + Spread

Spread (set in 6 months)

$$Spread = \left(\frac{98.5 * (5YearCMT)}{5.78} - P(30YearTbond) \right)$$

5Year CMT is the 5-year Constant Maturity Treasury rate set in 6 months and P(30yearTbond) is the price of the 6.25% coupon original 30-year Treasury bond maturing in August 2023 set in 6 months.

Spread can never be zero. **Spread expense = max(Spread,0).**

The Spread increases with interest rates:
As rates rise, the 5-yearCMT incresases and P(30-year T-bond) decreases.

With the Spread, the Swap contract that P&G made with BT has been converted from a "plain vanilla" swap to one with option features.

P&G Motives for this contract:

P&G states in court filings that they believed interest rates would remain stable with a slight rise over several years and that they targeted a floating rate at about 40 bp below the CP rate.

> **On November 4, 1993, the 5-year, constant maturity Treasury (CMT) was 5.033 percent (Chart 4) and the 5-year swap rate on that date was 5.288 percent, a 25.5 bp swap-spread over the T-bond rate. The implied 6-month forward rate quoted on Eurodollars (approximating the CP) was 3.80 percent and the spot 30-day CP rate was 3.15 percent.**

Pricing the Spread -- determining its expected present value:

The Spread was characterized by BT as the sale by P&G to BT of a put option on the 30-year T-bond at a strike price of 98.5. The premium for the option is the 75 bp reduction from the CP rate for the swap. It may also be characterized as a sale by P&G to BT of a cap depending on the values of the 5-year CMT and the 30-year T-bond price. Regardless of how characterized, the expected present value of the Spread depends upon these two stochastic values.

Valuing this option requires using a model of the term structure so that both the 5-year CMT and 30-year T-bond price can be modeled consistently and simultaneously. Based on the parameters estimated for the model, probabilities for different interest rate movements are determined and trees are formed for each of the instruments -- an interest rate for the 5-year CMT and and a T-bond price for the 30-year T-bond factoring in the price movement due to decreasing time to maturity. Data on the term structure on Nov.4, 1993 suggests that the Spread had a positive expected present value.

Pricing Interest Rate Option-Like Contracts (continued)

Outcomes of the P&G/BT 5 Year/30 Year Linked Swap Contract

1. Chart 1: Interest Rate Movements Before and After November 4, 1993

2. Chart 2: Interest Rate Volatilities for the 5-Year CMT and the 30-Year T-bond yield to maturities.
   a. Volatility was comparatively low in November 1993.
   b. Volatility rose as interest rates rose.

3. Chart 3: The Spread value skyrocketed shortly after the Federal Reserve raised the Federal Funds rate in early February 1994.

4. Chart 4: The term structure shifted up in a parallel manner as the Federal Reserve raised interest rates, with the 5-year CMT actually increasing more than the short-term rates and the 30-year T-bond increasing almost as much. By January 18, 1995, the term structure has flattened with the 5-year CMT and 30-year CMT almost equal.

5. Table 2: Interest rates would have to fall considerably from present levels to have the Spread value less than the .75 percent reduction in the CP rate. The present 5-year CMT is about 7.7 percent and the 30-year CMT is about 7.8 percent. At these rates, the put option is way in the money.

Measuring Risk of A Portfolio

Various Concepts

1. Individual portfolio element measures of risk versus entire portfolio measures of risk.
The entire portfolio risk is what needs to be measured for regulatory purposes.

2. Worst Case Scenario Measures -- entire portfolio measures versus adding elements.
These lead to extremes and incomparable measures because of incomparable nature of "worst case".

3. Duration measures of risk -- sensitivity of the portfolio to interest rate changes.
Ignores changes in value arising from the passage of time, volatility of interest rates and other prices, and changes in correlations among portfolio element values.

4. Value at Risk is a measure of market risk.
VaR measures the maximum estimated losses in market value of a position or portfolio that can be expected to be incurred until the position can be liquidated or neutralized. VaR employs measures of the probability that the portfolio will lose value over a specified horizon. For example, a value at risk of $10 million over a one-week horizon with a 95 percent confidence level is interpreted that in 5 percent of the weeks, on average, the portfolio could lose $10 million or more.

5. Value at risk does not incorporate:
Credit risk
Operations risk
Liquidity risk (except that the length of time to unwind certain types of positions may be due to illiquidity in the market)

Measuring Risk of A Portfolio (continued)

JP Morgan's RiskMetrics™ (JP Morgan, 1994)

> **A methodology that attempts to measure financial market risks consistently over entire positions in major asset classes using standard financial portfolio concepts.**

FIGURE 4
The Payoff to Buying a One-Period Interest Rate Cap

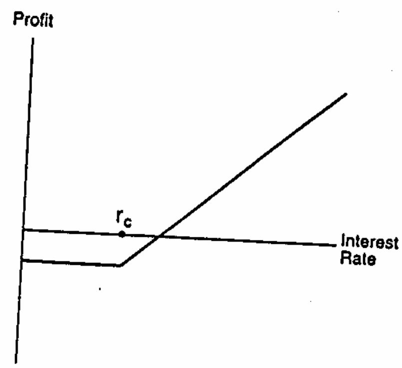


FIGURE 5
The Effect of Buying a Cap on Interest Expense

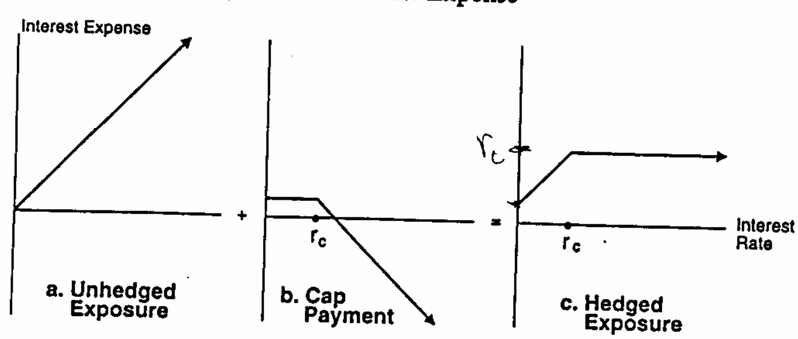


FIGURE 6
The Payoff to Buying a
One-Period Interest Rate Floor

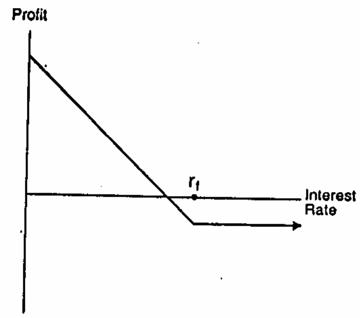


FIGURE 7
The Payoff to Buying a
One-Period, Zero-Cost Collar

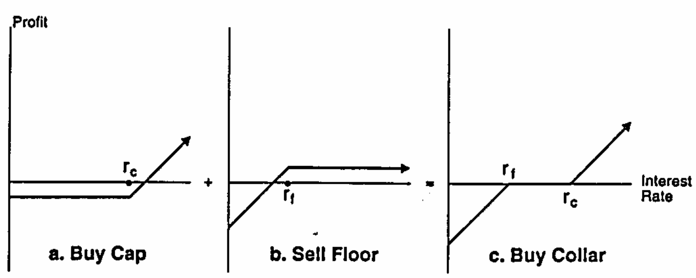
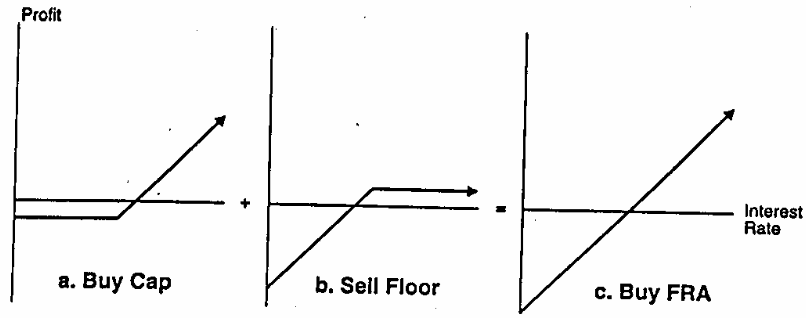
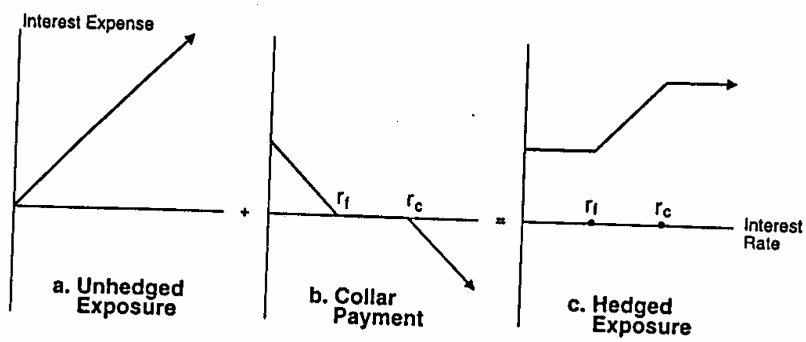


FIGURE 8
Put-Call Parity



FRA: Forward Rate Agreement

FIGURE 9
The Effect of Buying an Interest Rate Collar on Interest Expense



1.3.2 Risks associated with a portfolio of assets

To calculate the risks of a portfolio of assets, we need to have some understanding of **how prices move in relation to each other.** Market risk is additive only if instruments always move in a fixed relationship, i.e., their **correlation** is 1. Because correlations are not 1, we need to estimate them as we estimate volatilities.

The standard formula for estimating the risk of a two-asset portfolio is:

(B.1.3.1) $$\sigma_{AB} = \sqrt{a^2\sigma_A^2 + (1-a)^2\sigma_B^2 + 2a(1-a)\rho_{AB}\sigma_A\sigma_B}$$

where:

σ_A, σ_B = standard deviation of each asset

ρ_{AB} = correlation between assets A and B

a = proportion of position invested in asset A

The risks of the two-position portfolio are additive only if the correlation is equal to zero, in which case the third term of the equation above is eliminated.

Tables and Charts